

M.Sc. (Mathematics)

Semester-4

Full Marks: 70

Time: 3 Hours

[RING & FIELD]

Answer any five questions in which Q.No.-1 is compulsory.

The figures in the right hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Group-A
(Compulsory)

Each part of question carries 1 mark

1. (i) If in a UFD R , a, b are relatively prime then $a|bc$ implies $1 \times 10 = 10$
 \checkmark (a) $a|c$ (b) $a \nmid c$ (c) $c|a$ (d) None of these.
- (ii) If R be a UFD, then in $R[x]$ the product of two primitive polynomials is a
 (a) non-primitive polynomial
 \checkmark (b) primitive polynomial
 (c) constant (d) None of these
- (iii) If R is an integral domain with unity, then units of R and $R[x]$ are
 \checkmark (a) same (b) different (c) no relation between R and $R[x]$ units (d) None of these

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~~Q.10) $f(x) = x^3 - 9$ is reducible~~

(iv) $f(x) = x^3 - 9$ is

(a) irreducible in \mathbb{Z}_{11} (b) reducible in \mathbb{Z}_{11}

(c) not defined (d) None of these.

(v) Every non-zero, non unit element in a PID R is divisible by a

~~(a)~~ irreducible element (b) reducible element

(c) Not defined

(d) None of these.

(vi) If $f(x) \in R[x]$ is both primitive and irreducible element of $R[x]$ then $f(x)$ is

(a) reducible element of $K[x]$

☒ (b) irreducible element of $K[x]$

(c) $R[x] = K[x]$

(d) None of these.

(vii) Decomposition fields are

☒ (a) algebraic extensions

(b) normal extensions

(c) (a) & (b) both

(d) None of these.

(viii) Any field of characteristic 0 is

☒ (a) perfect (b) not perfect (c) not definite (d) None of these

(ix) The splitting field of $x^4 + 1 \in \mathbb{Q}[x]$ is $\mathbb{Q}(\alpha)$

where $\alpha = \cos \frac{\pi}{4} + i \sin \frac{\pi}{4}$ and

☒ (a) $[\mathbb{Q}(\alpha) : \mathbb{Q}] = 4$ (b) $[\mathbb{Q}(\alpha) : \mathbb{Q}] = \frac{\pi}{4}$ (c) 4 and $\frac{\pi}{4}$

(d) None of these

(x) Every ideal of a Euclidean ring is a

☒ (a) principal ideal (b) not a principal ideal (c) not defined

(d) None of these.

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Group - B

Answer any four questions

Each question carries 15 marks

4X15=60

2. (a) State and prove Eisenstein's theorem.

(b) In a U.F.D. (Unique factorization domain) [8]
 every pair of non-zero elements has a
 H.C.F. (highest common factor) and L.C.M.
 (lowest common multiple) [7]

3 (a) Find g.c.d. of the following polynomials
 under modulo 5,

$$f(x) = x^3 + 2x^2 + 3x + 2$$

$$g(x) = x^2 + 4$$

and express it as a linear combination
 of $f(x)$ and $g(x)$ [5]

(b) Every field is an Euclidean ring [5]

(c) Find the degree of minimal splitting field
 of $x^6 + 1$ over \mathbb{Q} . [5]

4 (a) Prove that an ideal S of a Euclidean
 ring R is maximal iff S is generated
 by some prime element of R [8]

(b) The necessary and sufficient condition
 that a non-zero element 'a' in a
 Euclidean ring R is a unit is that
 $d(a) = d(1)$ [7]

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Group-B

5. (a) An ideal S of the Euclidean ring R is maximal iff S is generated by some prime element of R . [8]

(b) The ring of polynomials over a field is a Euclidean. [7]

6. (a) Suppose a and b are arbitrary non-zero elements of a Euclidean ring R . Then show that

(i) If b is a unit of R , then $d(ab) = d(a)$ [5]

(ii) If b is not a unit of R , then $d(ab) > d(a)$ [5]

(b) Find the greatest common divisor of $(2+3i)$ and $(1+2i)$ in the ring of Gaussian integers. [5]

7. (a) The polynomial ring $\mathbb{Z}[x]$ over the ring of integers is not a principal ideal. [8]

(b) The polynomial domain $F[x]$ over a field F is a principal ideal domain. [7]

8. (a) Give an example of a ring in which some prime ideal is not a maximal ideal. [7]

(b) The field of all algebraic numbers is algebraically complete. [8]

SET-C

- Semester - 4

RING & FIELD

Answer of objective [Computing part - group-A]
question.

- Q1. (i) (a) (ii) (b) (iii) (a) (iv) (b) (v) (a)
(vi) (b) (vii) (a) (viii) (a) (ix) (a) (x) (a)