

M.Sc. (MATHEMATICS)

Semester - 4

[RING & FIELD]

Full Marks: 70

Time: 3 Hours.

Answer any five questions in which Q.No.-1 is compulsory.

The figures in the right hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A
(Compulsory)

Each part of question carries 1 mark.

1. (i) Every field is

- ☒ (a) an Euclidean ring (b) not an Euclidean ring
(c) (a) and (b) are conditional (d) none of these.

(ii) Every Euclidean ring is

- ☒ (a) principal ideal ring (b) not a principal ideal ring
(c) may or may not be principal ideal ring (d) none of these.

(iii) $x^2 - 3x + 5$

- (a) not primitive (b) primitive
(c) not necessarily primitive
(d) None of these

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- (iv) The ring of polynomials over a field is a
☒ (a) Euclidean ring (b) Not a Euclidean ring
 (c) It varies with conditions (d) None of these.
- (v) Decomposition fields are
 (a) normal extension ☒ (b) algebraic extensions
 (c) separable extension (d) None of these.
- (vi) Every finite ~~sp~~ ~~sp~~ separable extension is
 (a) complex ☒ (b) simple extension
 (c) simple and algebraic extension
 (d) None of these.
- (vii) The necessary and sufficient condition that a non-zero element a in a Euclidean ring R is a unit that
☒ (a) $d(a) = d(1)$ (b) $d(a) \neq d(1)$ ☒ (c) $d(a) = d(2)$
 (d) None of these.
- (viii) Any two finite fields having the same number of elements are
 (a) homomorphic ☒ (b) isomorphic
 (c) neither homomorphic nor isomorphic
 (d) None of these.
- (ix) The characteristic of a finite field is necessarily a
☒ (a) prime number (b) even number
 (c) odd number (d) None of these.
- (x) Any field of characteristic 0 is
☒ (a) perfect (b) not perfect (c) not definite (d) None of these

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Group-B

Answer any four questions

Each question carries 15 marks.

$$4 \times 15 = 60$$

- Q2. (a) State and prove Gauss Lemma on field of quotients of a unique factorization domain. [8]
- (b) Prove that the product of two primitive polynomials over a U.F.D. (Unique factorization domain) R is a primitive polynomial in $R[x]$. [7]
- Q3. (a) If p is prime and $f(x) = x^{p-1} + x^{p-2} + \dots + x + 1$, then $f(x)$ is irreducible over R , the field of real numbers. [7]
- (b) Every ideal of a Euclidean ring is a principal ideal. [8]
- Q4. (a) Let a and b be arbitrary elements of a Euclidean ring R , such that at least one of them is non-zero. Then there exists a g.c.d. $d \in R$ such that $d = ma + nb$ for some $m, n \in R$. [8]
- (b) Find the g.c.d. of the following polynomials: $f(x) = x^3 + \frac{1}{2}x^2 + \frac{1}{3}x + \frac{1}{6}$ and $g(x) = x^2 - \frac{1}{2}x - \frac{1}{2}$ and express it as linear combination of two polynomials. [7]

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Q5. The ring of Gaussian integers is an Euclidean domain (or Euclidean ring).

Q6. (a) Any finite extension of a field is an ~~extension~~ algebraic extension of the field. [7]

(b) Let K be a finite algebraic extension of a field F . Then K is a normal extension of F iff K is the splitting field over F of some non-zero polynomial over F . [8]

Q7. (a) Any finite extension of a field of characteristic 0 is a simple extension. Prove this. [8]

(b) Every finite separable extension is simple. [7]

Q8. K is a normal extension of a field F of characteristic 0 iff K is the splitting field of some polynomial over F . [15]

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[RING & FIELD]

Answers of Objective questions

Group - A [Compulsory] - Q. No. - 1

1. (i) (a) (ii) (a) (iii) (b) (iv) (a) (v) (b)
(vi) (b) (vii) (a) (viii) (b) (ix) (a) (x) (a)