SET–A

M.Sc. (MATHEMATICS)

Semester – 4

[RING & FIELD]

Full Marks: 70

Time: 3 Hours.

Answer any five question in which Q.No.–1 is compulsory.

The figures in the right hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

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Group – A
(Compulsory)

Each part of question carries 1 mark.

1X10=10

1. (a) The ring of Gaussian integers is
   (i) not an Euclidean domain
   (ii) an Euclidean domain
   (iii) may or may not be Euclidean domain
   (iv) None of these.

(b) An Euclidean ring R is:
   (i) Principal ideal ring
   (ii) Not a principal ideal ring
   (iii) (i) & (ii) are conditional

(c) An element $a \in F$ is a zero of $f(x) \in F[x]$ is iff $(x-a)$ is
   (i) a factor of $f(x)$ (ii) is not a factor of $f(x)$
   (iii) a substitute of $f(x)$ (iv) None of these

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(d) Any two elements a, b of an integral domain R are said to be relatively prime or coprime if their g.c.d. is

(i) 2 (ii) 1 (iii) 1 or 2 (iv) None of these.

(e) A polynomial $f(x) \in R[x]$ is called a primitive if the g.c.d. of the coefficients is

(i) 1 (ii) any odd number (iii) 2

(iv) None of these

(f) An element $a (\neq 0)$ of an integral domain R is said to be a divisor or factor of an element $b \in R$ if $\exists$ $c \in R$ such that

(i) $b = \frac{a}{c}$ (ii) $b = ac$ (iii) $b = a + c$ (iv) None of these

(g) $x^3 - 2 \in Q[x]$ is irreducible as it has

(i) no root in Q (ii) roots in Q

(iii) No connection with roots (iv) None of these.

(h) $f(x) = x^2 - 5x + 6$ is

(i) seperable over Q (ii) not seperable over Q

(iii) depends upon the value of x (iv) None of these

(i) A field F is called perfect if all finite extensions of F are

(i) not seperable (ii) seperable (iii) depends upon other basic conditions (iv) None of these

(j) The relation of divisibility in an integral domain R is

(i) reflexive (ii) transitive (iii) reflexive and transitive

(iv) None of these.

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Group - B

Answer any four questions

Each question carries 15 marks

$4 \times 15 = 60$

2. (a) What is unique factorisation domain? Show that the ring of polynomials in 'x' with coefficients from a field form a unique factorisation domain. [10]

(b) Find the g.c.d of the polynomials
$f(x) = x^5 + 2x^3 + x^2 + 2x$, $g(x) = x^4 + x^3 + x^2$
over the field of modulo 3. [5]

3. (a) Show that the polynomial [7]
$x^4 + x^3 + x^2 + x + 1$ is irreducible over Q.

(b) State and prove the Unique Factorization Theorem. [8]

4. (a) An ideal $S = (a_0)$ is maximal ideal of the Euclidean ring R iff a_0 is a prime element of R. [10]

(b) Find the maximal ideals of the ring of integers [5]

5. (a) State and prove Unique Factorisation Theorem. [8]

(b) Show that ring of integers is a Euclidean ring. [7]

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6. Let K be finite extension of a field F. Then the group $G(K,F)$ of automorphisms of K relative to F is a finite group and its order, $O(G(K,F))$ satisfies the relation
$$O[G(K,F)] \leq [K:F]$$
[15]

7. Suppose K is the field of complex numbers and F is the field of real numbers, show that K is a normal extension of F. [15]

8. (a) Let K be a finite algebraic extension of a field F. Then K is a normal extension of F iff K is the splitting field over F, of some non-zero polynomial over F. [8]

(b) Let a, b, c be roots of x^3-3x+1 in $\mathbb{C}$. Show that $\mathbb{Q}(a)$ is a normal extension of $\mathbb{Q}$. [7]

Answers of objective question [Q. No. - 1]

1.
(a) (ii) (b) (i) (c) (i) (d) (ii) (e) (i)
(f) (ii) (g) (i) (h) (i) (i) (ii) (j) (iii)