MODAL QUESTIONS

Set – one

P.G. sem three, paper – CC-309

PREPARED
BY
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for Kolhan university, Chaibasa.

Time : 3 Hrs.

FULL MARKS : 70

Instruction: Candidates are required to give their answers in their own words as far as practible.
The figure indicates in the margin for full marks.

PART A

Q.No. 1. Answer all questions: $1 \times 10 = 10$

(a) If $F(t) = t^2$ is of exponential order of
(i) 0 (ii) 1 (iii) 2 (iv) 3

(b) If $L\left\{\frac{\sin at}{t}\right\} = \frac{s}{s^2+a^2}$, then $L\left\{\frac{\cos at}{t}\right\}$
(i) $\frac{1}{s^2+a^2}$ (ii) $-\frac{s}{s^2+a^2}$ (iii) exists (iv) does not exist.

(c) $L^{-1}\left\{\frac{s-a}{(s-a)^2+b^2}\right\}$ is equal to

(i) $e^{at}\sin bt$ (ii) $e^{at}\cos bt$ (iii) $e^{at}\tan bt$ (iv) $e^{at}\sec bt$

(d) $L\{y'(t)\}$ is equal to

(i) $py(p)-y(0)$ (ii) $p\bar{y}(p)-y(0)$

(iii) $p\bar{y}(p)-\bar{y}(0)$ (iv) $py(p)-\bar{y}(0)$

(e) Let $F(x)$ be a function defined on $(-\infty,\infty)$ and piecewise continuous in each partial interval in $(-\infty,\infty)$. Then

$$F\{F(x)\} = f(s) = \int_{-\infty}^{\infty} e^{isx} F(x)\,dx \text{ is called}$$

(i) Fourier transform of $F(x)$

(ii) Laplace transform of $F(x)$

(iii) Hankel tranform of $F(x)$

(iv) None of these

(f) Fourier sine transform of $F(x)=\frac{1}{x}$ is

(i) π (ii) $-\frac{\pi}{2}$ (iii) $\frac{\pi}{2}$ (iv) $-\pi$.

(g) If $0 < F(x) < l$ and $n \geq 0$ then

$f_c\{F(x)\} = F_c(x) =$

(i) $\int_0^l F(x)\cos\frac{n\pi x}{l}\,dx$ (ii) $\int_l^{\infty} F(x)\cos\frac{n\pi x}{l}\,dx$

(iii) $\int_0^{\infty} F(x)\frac{\cos n\pi x}{l}\,dx$ (iv) none of these

(h) If $x = x_0$ is an ordinary point of the equation $P_0(x)\frac{d^n y}{dx^n} + P_1(x)\frac{d^{n-1} y}{dx^{n-1}} + \cdots + P_n(x)y = 0$, if $P(x_0)$ does not vanish for $x = x_0$. Then $x = x_0$ will be a singular point, if

(i) $P_0(x_0) = 1$, for $x = x_0$ (ii) $P_0(x_0) = 0$, for $x = x_0$

(iii) $P_0(x_0) = -1$, for $x = x_0$ (iv) $P_0(x_0) = \infty$, for $x = x_0$.

(i) The characteristic vectors corresponding to distinct characteristic roots of a matrix are
(i) equal (ii) unequal

(iii) linearly dependent (iv) linearly independent.

(J) A is a symmetric matrix, then Eigen values are

(i) repeated (ii) non-repeated (iii) imaginary (iv) none.

PART B

Answer any four questions of the following $15 \times 4 = 60$

Q.No. (2) (a) Find $L\{t^3 \sin t\}$ and hence evaluate
$$\int_0^\infty t^3 e^{-t} \sin t \, dt.$$

(b) If $F(t) = t^2$, $0 < t < 2$ and
$F(t+2) = F(t)$, find $L\{F(t)\}$

Q.No (3) (a) Apply Heaviside's expansion formula to find $L^{-1}\left\{\dfrac{6p^2 + 22p + 18}{p^3 + 6p^2 + 11p + 6}\right\}$.

(b) Apply Convolution theorem, find
$$L^{-1}\left\{\frac{p^2}{(p^2+4)^2}\right\}.$$

Q.No (4) Solve $(D-2)x - (D+1)y = 6e^{3t}$,
$(2D-3)x + (D+3)y = 6e^{3t}$,
if $x = 3$, $y = 0$ when $t = 0$

Q. No. (5) Solve $ty'' + 2y' + ty = 0$ if $y(0) = 1$, $y(\pi) = 0$

Q. No. (6) (a) Find $L\{t\, J_0(at)\}$

(b) Find the Fourier sine and cosine transforms of $\dfrac{e^{ax} + e^{-ax}}{e^{\pi x} - e^{-\pi x}}$.

Q. No. (7) Determine the solution of the equation in series
$$3x\frac{d^2y}{dx^2} + (2-x)\frac{dy}{dx} - y = 0$$

Q.NO. 8 Find the eigen values and the corresponding eigen vectors for the matrix $A = \begin{bmatrix} 9 & -1 & 9 \\ 3 & -1 & 3 \\ -7 & 1 & -7 \end{bmatrix}$

Q.NO. 9 Solve $\frac{\partial y}{\partial t} = 3\left(\frac{\partial^2 y}{\partial x^2}\right)$, $y_x(0,t) = 0$, $y\left(\frac{\pi}{2}, t\right) = 0$ and $y(x,0) = 20\cos 3x - 5\cos 9x$.

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Answers of objective questions

Q. NO. 1

a — (iv)
b — (iv)
c — (ii)
d — (ii)
e — (i)
f — (iii)
g — (i)
h — (ii)
i — (iv)
j — (ii)

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K.C.C.

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MODAL QUESTIONS

SET – TWO

P.G. Sem III, Paper – CC-309

PREPARED

BY

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JAMSHEDPUR

for, Kolhan university, Chaibasa.

Time : 3 Hrs.

FULL MARKS : 70

Instruction: Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

PART – A

Q.No. (1) Answer all questions : 1×10 = 10

(a) $L\{e^{at}t^n\}$ is equal to

(i) $\frac{\lfloor n}{s+a}$ (ii) $\frac{\lfloor n}{(s-a)^{n+1}}$

(iii) $\frac{\lfloor n}{(s-a)^n}$ (iv) none of these

(b) If $L\{F(t)\} = f(s)$, then $L\left\{\frac{F(t)}{t}\right\}$ is =

(i) $\int_0^\infty f(x)\,dx$ (ii) $\int_1^\infty f(x)\,dx$

(iii) $\int_s^\infty f(x)\,dx$ (iv) none of these

(c) If $L^{-1}\{f(s)\} = F(t)$, then $L^{-1}\{f^{(n)}(s)\} =$

(i) $(-1)\,t^n F(t)$ (ii) $(-1)^n t^n F(t)$

(iii) $(-1)^{-n} t^n F(t)$ (iv) $(-1)^n t^n [F(t)]^n$

(d) $L\left\{\frac{\partial^2 y}{\partial x^2}\right\}$ is equal to

(i) $\frac{d^2\bar{y}}{dx^2}$ (ii) 0 (iii) 1 (iv) -1

(e) $\frac{d^2y}{dx^2} + y = 0$, under the conditions $y = 1$, $\frac{dy}{dt} = 0$ when $t = 0$ then $y =$

(i) $\sin t$ (ii) $\cos t$ (iii) $\tan t$ (iv) $\cot t$

(f) If $F\{f(x)\} = f(p)$, $F\{G(x)\} = g(p)$ be the convolution of $F(x)$ and $G(x)$, then

$F\{F(x) * G(x)\} =$

(i) $f(p) * g(p)$ (ii) $f(p)/g(p)$

(iii) $f(p) + g(p)$ (iv) $f(p) = g(p)$.

(g) The Fourier cosine transform of

$$f(x) = \begin{cases} 1, & 0 \le x < 1 \\ 0, & x > 1 \end{cases} \text{ is}$$

(i) $\frac{\sin s}{s}$ (ii) 1 (iii) $\frac{\cos s}{s}$ (iv) none of these

(h) The sum of the series

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \infty =$$

(i) $\frac{\pi^2}{12}$ (ii) $\frac{\pi^2}{8}$ (iii) $\frac{\pi^2}{6}$ (iv) $\frac{\pi^2}{2}$

(i) Roots of indicial equation unequal and differing by a quantity

(i) not an imaginary (ii) not an irrational
(iii) not an integer (iv) none of these

(J) Any two characteristic vectors corresponding to two distinct characteristic roots of a real symmetric matrix are

(i) unitary (ii) symmetric
(iii) Anti symmetric (iv) orthogonal.

PART – B

Answer any four questions of the following:

$15 \times 4 = 60$

Q. No. 2

(a) Find L.T. of $\int_0^t \left(\frac{1-e^{-2x}}{x}\right) dx$ 5

(b) Find L.T. of $\left(\frac{e^t - \cos t}{t}\right)$ 5

(c) Find L.T. of $\int_0^t \frac{\sin x}{x} dx$ 5

Q No. 3 State and prove the convolution theorem, and find $L^{-1}\left\{\frac{1}{(s-1)\sqrt{s}}\right\}$

Q No 4 (a) Solve the differential equation $tY''(t) + Y'(t) + tY(t) = 0$ under the conditions that $Y(0) = 1$ and $Y(t)$ and its derivatives have transforms.

(b) Apply Laplace transform to solve $(D^2+9)y = 6\cos(3t)$, $y(0) = 2$, $y'(0) = 0$.

Q No 5 (a) State and Prove the linearity property of fourier transform 5

(b) State and Prove the change of scale property of fourier transform. 5

(c) State and prove the shifting property of fourier transform. 5

Q. No. 6 (a) Find the Fourier Sine and Cosine transform of $x^n e^{-ax}$.

(b) Find the Fourier Sine transform of $\frac{x}{1+x^2}$

Q. No. 7 State and Prove Fourier integral theorem.

Q No. 8 Solve the equation
$$(1-x^2)\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} - 2y = 0$$ for which $x = 0$ is an ordinary point

Q No. 9 Find the Eigen values and the corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$$

——— X ———

Answers of objectives Q. No. 1

a — (i)	f — (ii)
b — (iii)	g — (i)
c — (ii)	h — (ii)
d — (i)	i — (iii)
e — (ii)	J — (iv)

——— X ———

MODAL QUESTIONS

Set - Three

P. G. Sem - three, paper - CC309

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FULL MARKS : 70

Instruction: Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

PART - A

Q. No. 1 Answer all questions: 1×10 = 10

(a) $L\{J_0(t)\}$ is equal to

(i) $\frac{1}{\sqrt{p^2+1}}$ (ii) $\frac{p}{\sqrt{p^2+1}}$ (iii) $-\frac{1}{\sqrt{p^2+1}}$ (iv) $-\frac{p}{\sqrt{p^2+1}}$

(b) $s^3 f(s) - s^2 F(0) - sF'(0) - F''(0) =$

(i) $L\{F''(t)\}$ (ii) $L\{F'(t)\}$

(iii) $L\{F'''(t)\}$ (iv) none of these

(c) $L^{-1}\left\{\frac{1}{(s-4)^5}+\frac{5}{(s-2)^2+5^2}\right\} =$

(i) $e^{2t}\left(\frac{t^4}{\lfloor 4}\right)+e^{2t}\sin 5t$

(ii) $e^{3t}\left(\frac{t^4}{\lfloor 4}\right)+e^{3t}\sin 5t$

(iii) $e^{4t}\left(\frac{t^4}{\lfloor 4}\right)+e^{2t}\sin 5t$

(iv) none of these.

(d) The cosine transform of e^{-x} is

(i) $\frac{p}{1+p^2}\sqrt{\frac{2}{\pi}}$ (ii) $\frac{1}{1+p^2}$

(iii) $\frac{p^2}{1+p^2}\sqrt{\frac{2}{\pi}}$ (iv) $\sqrt{\frac{2}{\pi}}$

(e) Inversion formula for the infinite Fourier transform is

(i) $F(x)=\frac{2}{\pi}\int_0^\infty f(p)\sin(px)\,dp$

(ii) $F(x)=\frac{1}{\pi}\int_0^\infty f(p)\sin(px)\,dp$

(iii) $F(x)=\frac{1}{\pi}\int_0^\infty f(p)\sin x\,dp$

(iv) None of these.

(f) The finite sine transform of $\frac{x}{\pi}$ is

(i) $\frac{(-1)^p}{p}$ (ii) $\frac{(-1)^{p-1}}{p^2}$ (iii) $\frac{(-1)^{p+1}}{p}$ (iv) $\frac{(-1)^{p+1}}{p^2}$

(g) For Fourier expansion of an odd function in $(-\pi, \pi)$

(i) a_0 is zero (ii) a_n is zero

(iii) b_n is zero (iv) both a_0 and a_n zero

(h) The roots of indicial equation are k_1 and k_2 and not differ by an integer. Then by Frobenius method, the general solution of the equation is

(i) $y = au + bv$, where a and b are arbitrary constants

(ii) $y' = au + bv$, where a and b are arbitrary constants

(iii) $y = au' + bv'$, where a and b are arbitrary constants

(iv) $y' = au' + bv'$, where a and b are arbitrary constants

(i) If $F(t) = t^2$ is of exponential order of

(i) 0 (ii) 1 (iii) 2 (iv) 3

(j) $L\left\{ \frac{\partial^2 y}{\partial x^2} \right\}$ is equal to

(i) $\frac{d^2 y}{dx^2}$ (ii) 0 (iii) 1 (iv) -1

PART – B

Answer any four questions of the following $15 \times 4 = 60$

Q. No. (2) (a) Let a function $F(t)$ be periodic with period w, so that $F(t + nw) = F(t)$, $n = 1, 2, 3, \ldots$ then prove that

$$L\{F(t)\} = \int_0^w e^{-st} F(t)\, dt \Big/ 1 - e^{-sw}$$

(b) Prove that

$$\lim_{t \to 0} F(t) = \lim_{s \to \infty} s f(s) \text{ and}$$

$$\lim_{t \to \infty} F(t) = \lim_{s \to 0} s f(s)$$ 5

(c) Find the Laplace transform of

$$f(t) = \begin{cases} \sin \omega t & \text{for } 0 < t < \pi/\omega \\ 0 & \text{for } \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$$ 5

Q No. (3) (a) Evaluate

$$L^{-1}\left\{ \frac{1}{(s-4)^5} + \frac{5}{(s-2)^2+5^2} + \frac{s+3}{(s+3)^2+6^2} \right\}$$ 5

(b) If $L^{-1}\{f(s)\} = F(t)$, then prove that

$L^{-1}\{e^{as} f(s)\} = G(t)$, where

$$G(t) = \begin{cases} F(t-a), & t > a \\ 0, & t < a \end{cases}$$ 5

(c) If $f(s) = L\{F(t)\}$ denotes the Laplace transform of the function $F(t)$, prove that $L^{-1}\{f(as)\} = \frac{1}{a} F\left(\frac{t}{a}\right)$, $a > 0$ 5

Q No. (4) Solve $\frac{d^2x}{dt^2} + \frac{dy}{dt} + 3x = 15e^{-t}$,

$$\frac{d^2y}{dt^2} - 4\frac{dx}{dt} + 3y = 15 \sin 2t.$$

if $x(0) = 35$, $x'(0) = -48$, $y(0) = 27$, $y'(0) = -55$.

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Q. No (5) (a) Establish the relation between Fourier transform and Laplace transform. 5

(b) Find the fourier transform of

$$F(x) = \begin{cases} 1-x^2, & |x|<1 \\ 0, & |x|>1 \end{cases}$$ 5

and hence evaluate

$$\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right) \cos\frac{x}{2}\, dx$$

(c) Find the complex fourier transform of $f(x) = e^{-a|x|}$, where $a>0$ and x belongs to $(-\infty, \infty)$ 5

Q. No (6) (a) Using fourier integral, show that

$$e^{-ax} = \frac{2a}{\pi}\int_0^\infty \frac{\cos\lambda x}{\lambda^2+a^2}\, d\lambda, \quad a>0, x\geq 0$$

(b) Find finite fourier sine and cosine ~~transform of $f(x)=x^2$, $0<x<4$.~~

Q. No. (7) (a) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ by F.T.

with conditions (i) $u(0,t)=0$,

(ii) $u = \begin{cases} 1, & 0<x<1 \\ 0, & x\geq 1 \end{cases}$ when $t=0$

(iii) $u(x,t)$ is bounded

(b) Reduce the differential equation
$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 6, \quad t > 0$$
to ordinary differential equation by using finite fourier transform.

Q.No 8 Solve in series
$$9x(1-x)y'' - 12y' + 4y = 0$$
using frobenius method.

Q. No 9 Find the Eigen vectors of the matrix
$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

— x — x —

Answers of objectives Q. No. 1

a — (i)
b — (iii)
c — (iii)
d — (ii)
e — (i)
f — (iii)
g — (iv)
h — (i)
i — (iv)
j — (i)

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