University Deptt. of Mathematics
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M. Sc: 3rd Semester
Paper code: ECMATH 302 B
Paper : Difference Equation

SET: C

Answer form all the section as directed
Q. No. 1 is compulsory

The figures in the right-hand margin indicate marks.

Candidates are required to give their answer in their own words as far as practicable.

SECTION - I
(compulsory)

1. Choose the correct option of the following: 1 × 10

<a> If the difference of interval is k then $\Delta^{-1}(k) =$ ______

<i> $1/x$ ii> x iii> 0 iv> 1

<b> The particular soln of the difference equation

$$9y_{x+2} - 6y_{x+1} + y_x = 0$$

is ______ when $y_0 = 0$ and $y_1 = 1$

i> $y_x = (1/3)^x$ ii> $y_x = x(1/3)^x$

iii> $y_x = 3x(1/3)^x$ iv> None

(c) General solution of

$y_{x+1} = -y_x + 3$; $y_0 = 6$

over the set of $x = 0, 1, 2, \ldots$

<i> $y_x = \frac{3}{2} + \frac{9}{2}(-1)^x$

<ii> $y_x = \frac{3}{2} - \frac{9}{2}(-1)^x$

iii> $y_x = -\frac{3}{2} + \frac{9}{2}(-1)^x$

<iv> None

<d> If $y(a) = x+5$ and $k=1$ then

$\Delta y(1) =$ ______

<i> 1 ii> 13 iii> 0 iv> 4

<e> A point b is said to be eventually k- periodic point of $f(x)$ if for some positive Integer m, we have

<i> $f^m(b) = f^k(b)$

ii> $f^{m+k}(b) = f^m(b)$

<iii> $f^{m+k}(b) = f^k(b)$

iv> $f^{mk}(b) = f(b)$

<f> If x^* be equilibrium point of difference equation $x(n+1) = f(x(n))$; where f is continuously differenti- -able at x^*; then x^* is an asymptotically stable point if

<i> $|f'(x^*)| < 1$ <ii> $|f'(x^*)| > 1$

iii> $|f'(x^*)| < 0$ iv> $|f'(x^*)| > 0$

<g> The no. of equilibrium points of eqn

$$x(n+1) = x^3(n)$$

is ________

<i> 0 ii> 3 iii> 2 iv> 1.

<h> A point b is said to be periodic point of $f(x)$ if for some integer k, we have

<i> $f^k(b) = b$ ii> $f^{m+k}(b) = f^m(b)$

iii> $f(b) = b$ iv> $f^{mk}(b) = b$

<i> General solution of

$$y_{n+4} - 4y_{n+3} + 6y_{n+2} - 4y_{n+1} + y_n = 0$$

is ________

<i> $y_n = c_1 + c_2 n + c_3 n^2 + c_4 n^3$

ii> $y_n = c_1 + c_2 2^n + c_3 3^n + c_4 4^n$

iii> $y_n = (c_1 + c_2 n) + (c_3 + c_4 n) 2^n$

iv> None.

j> $\Delta \log f(x) =$ ________

<i> $\log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$

<ii> $\log\left[\frac{\Delta f(x)}{f(x)}\right]$

iii> $\log\left[1 - \frac{\Delta f(x)}{f(x)}\right]$

iv> None.

SECTION: II

Answer any four questions (15×4)

<2> <a> Define difference equation, order and degree of difference equation with some examples.

<b> Eliminate the arbitrary constants A and B from $y_n = A.2^n + B.3^n$ and derive the corresponding difference equation.

<3> Show that the function $y_k = 1 - \frac{2}{k}$ $k = 1, 2, 3 \ldots$ is a sol^n of difference equation $(k+1) y_{k+1} + k y_k = 2k - 3$

<4> State and prove Fundamental theorem for difference Calculus.

<5> Solve: $y_{k+2} - 4y_{k+1} + 4y_k = 3k + 2^k$

<6> Prove that the solution of the difference eq^n $y_{k+1} = Ay_k + B$, $k = 0, 1, 2, 3, \ldots$ where A and B are constants is given by

$$y_k = \begin{cases} A^k y_0 + \dfrac{B(1-A^k)}{1-A} & ; \text{if } A \neq 1 \\ y_0 + Bk & ; \text{if } A = 1 \end{cases}$$

<7> Solve:

<a> $y_{k+4} - 8y_{k+3} + 18y_{k+2} - 27y_k = 0$

<b> $y_{x+2} + y_x = 0$; $y_0 = 1$. and $y_1 = 1$

<8> Solve $U_{x+4} + U_x = 0$

or

Apply matrix method to solve
$x_{t+1} - 3x_t + 2x_{t-1} = 0; \ t = 1, 2, 3 \ldots$

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Answer key

SET : C

Q. No. corresponding ans,

1. <a> ———— <ii>

<b> ———— <iii>

<c> ———— <i>

<d> ———— <i>

<e> ———— <ii>

<f> ———— <i>

<g> ———— <ii>

<h> ———— <i>

<i> ———— <i>

<j> ———— <i>