University Deptt. of Mathematics
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M.Sc: 3rd semester
Papercode: ECMATH302 B
paper: Difference Equation

SET: B

Answer form all the section as directed.
Q.No. 1 is compulsory.
The figures in the right-hand margin indicate marks.
Candidates are required to give their answers in their own words as far as practicable.

SECTION - I
(compulsory)

1. Choose the correct option of the following: 1 × 10

<a> The order and degree of the difference equation

$$y_{k+3} + y_{k+2} - y_{k+1} - y_k = 0$$

respectively are

i> 1, 3 ii> 3, 1 iii> 2, 3 iv> 3, 2

<b> The solution of the difference eqn $y_{x+1} - y_x = x$ is ________

<i> $y_x = \frac{x(x+1)}{2}$ <ii> $y_x = \frac{x-1}{2}$

iii> $y_x = \frac{x(x-1)}{2}$ iv> $y_x = \frac{x+1}{2}$

<c> The no. of equilibrium points of
$$x(n+1) = x^2(n) - x(n) + 1$$
is equal to ______

<i> 3 ii> 1 iii> 0 iv> 2

<d> General soln of $y_{h+3} + y_{h+2} - 8y_{h+1} - 12y_h = 0$ is

<i> $y_h = c_1 3^h + (c_2 + c_3 h)(-2)^h$

<ii> $y_h = c_1 2^h + (c_2 + c_3 h)(-3)^h$

<iii> $y_h = c_1 3^h + c_2 2^h + c_3 (-2)^h$

<iv> None.

<e> If $y(x) = x^2 + 2$ and $k = 1$ then
$\Delta y(1.5) =$ ______

i> 4 ii> 3 iii> 2 iv> 1.6

<f> The solution of difference equation
$$x(n+1) = \alpha(n)\, x(n)$$
where $x(n_0) = x_0$; $n \geq n_0$
is ______

<i> $x(n) = \left[\prod_{i=n_0} \alpha(i)\right] x_0$

<ii> $x(n) = \left[\prod_{i=n_0}^{n-1} \alpha(i)\right] x_0$

<iii> $x(n) = \left[\prod_{i=n_0}^{n-2} \alpha(i)\right] x_0$

<iv> $x(n) = \left[\prod_{i=n_0}^{n+1} \alpha(i)\right] x_0$

<g> The auxiliary equation of difference eqn $3y_{x+2} - 6y_{x+1} + 7y_x = 0$ is ______

<i> $3m^2 - 6m + 7 = 0$

<ii> $3m^2 - 6m - 7 = 0$

iii> $3m^2 + 6m - 7 = 0$

iv> None.

<h> A point b is said to be eventually k-periodic point of $f(x)$ if for some positive integer m, we have ______

<i> $f^{mk}(b) = f(b)$

ii> $f^{m+k}(b) = f^k(b)$

iii> $f^{m+k}(b) = f^m(b)$

iv> $f^m(b) = f^k(b)$

i> The value of $\Delta^5 x^{(4)}$ for the interval of difference k is ______

<i> 0 ii> $\lfloor 5 \, k^5$ iii> $\lfloor 5 \, k^4 x$ iv> $\lfloor 5 \, k^5 x$

j> A point x^* in the domain of f is said to be an equilibrium point of eqn $x(n+1) = f(x(n))$ if

<i> $f(x^*) = x^*$

<ii> $f(x^*) = 2x^*$

iii> $f(x^*) = 3x^*$

iv> $f(x^*) = 0$

SECTION – II

Answer any four questions (15×4)

<2> <a> Find the order and degree of the equations

i> $\Delta^3 y_x + 2\Delta y_x + y_x = x+3$

ii> $y_{x+4} - 5y_{x+2} + 6y_x = 0$

<b> Show that $y_x = 1 - \frac{2}{x}$, $x = 1, 2, 3, \ldots$ is a solution of the first order difference equation

$(x+1)y_{x+1} + xy_x = 2x-3$, $x = 1, 2, 3, \ldots$

<3> <a> Define the periodic point, eventually periodic point and periodic orbit.

<b> Prove that

$y_x = y_{x-1} + \Delta y_{x-2} + \Delta^2 y_{x-3} + \ldots + \Delta^n y_{x-n} + \Delta^n y_{x-n}$

<4> State and prove fundamental theorem for difference calculus.

<5> Solve

<a> $y_{k+4} - 8y_{k+3} + 18y_{k+2} - 27y_k = 0$

<b> $y_{x+2} + y_x = 0$; with $y_0 = 1$, $y_1 = 1$

<6> Discuss the cobweb phenomenon with diagrams.

<7> If y_k satisfies the difference eqn

$y_{k+1} - \lambda y_k + y_{k-1} = 0$, $k = 1, 2, 3.$

and the end conditions $y_0 = y_4 = 0$, determine λ on which a nontrivial solution exists.

<8> Solve $y_{k+2} - 6y_{k+1} + 8y_k = 3k^2 + 2 - 5 \cdot 3^k$ by the method of undetermined coefficients.

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Answer key

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Q. No.		corresponding ans.
1.	<a>	<ii>
	<b>	<iii>
	<c>	<ii>
	<d>	<i>
	<e>	<i>
	<f>	<ii>
	<g>	<i>
	<h>	<iii>
	<i>	<i>
	<j>	<i>